7. Appendix (Proofs of Statements)

Proof [Theorem 3] We first demonstrate that, under condition (9), $\mathcal{B}$ constructed by solving SOP$_{phy}$ in (7) satisfies (3c) across the entire range of X. Note that according to (8), for any $x \in X$, there exists $\hat{x}^p \in \mathsf{X}^p$ such that x and $\hat{x}^p$ are ϵ^p-close. Since $\mathcal{B}(q^*, f(x)) - \kappa^*\mathcal{B}(q^*, x)$ is Lipschitz continuous with respect to x with Lipschitz constant $\mathscr{L}^2$, and by adding and subtracting $\mathcal{B}(q^*, f(\hat{x}^p)) - \kappa^*\mathcal{B}(q^*, \hat{x}^p)$, one has

$$\begin{aligned}\mathcal{B}(q^*, f(x)) - \kappa^*\mathcal{B}(q^*, x) &= \mathcal{B}(q^*, f(x)) - \kappa^*\mathcal{B}(q^*, x) - \big(\mathcal{B}(q^*, f(\hat{x}^p)) - \kappa^*\mathcal{B}(q^*, \hat{x}^p)\big)\\ &\quad + \underbrace{\big(\mathcal{B}(q^*, f(\hat{x}^p)) - \kappa^*\mathcal{B}(q^*, \hat{x}^p)\big)}_{\leq \eta^*_{phy}}\\ &\leq \mathscr{L}^2\|x - \hat{x}^p\| + \eta^*_{phy} \leq \mathscr{L}\|x - \hat{x}^p\| + \eta^*_{phy} \overset{(8)}{\leq} \underbrace{\mathscr{L}\epsilon^{\max} + \eta^*_{phy}}_{=\eta^*_R} \overset{(9)}{\leq} 0.\end{aligned}$$

Thus, by defining $\eta^*_R = \mathscr{L}\epsilon^{\max} + \eta^*_{phy}$, the constructed $\mathcal{B}$ obtained by solving the SOP$_{phy}$ in (7) satisfies (3c) over the entire range of X. By employing similar reasoning and adding and subtracting $\mathcal{B}(q^*, \hat{x}^p)$, one can demonstrate that under condition (9), the constructed $\mathcal{B}$ resulting from solving the SOP$_{phy}$ in (7) fulfills conditions (3a) and (3b) for the range of X_0 and X_u, respectively:

$$\begin{aligned}\mathcal{B}(q^*, x) - \alpha^* &= \mathcal{B}(q^*, x) - \mathcal{B}(q^*, \hat{x}^p) + \underbrace{\mathcal{B}(q^*, \hat{x}^p) - \alpha^*}_{\leq \eta^*_{phy}} \leq \mathscr{L}^1\|x - \hat{x}^p\| + \eta^*_{phy}\\ &\leq \mathscr{L}\|x - \hat{x}^p\| + \eta^*_{phy} \overset{(8)}{\leq} \underbrace{\mathscr{L}\epsilon^{\max} + \eta^*_{phy}}_{=\eta^*_R} \overset{(9)}{\leq} 0, \ \text{ and}\\ -\mathcal{B}(q^*, x) + \rho^* &= \mathcal{B}(q^*, \hat{x}^p) - \mathcal{B}(q^*, x) \underbrace{-\mathcal{B}(q^*, \hat{x}^p) + \rho^*}_{\leq \eta^*_{phy}} \leq \mathscr{L}^1\|x - \hat{x}^p\| + \eta^*_{phy}\\ &\leq \mathscr{L}\|x - \hat{x}^p\| + \eta^*_{phy} \overset{(8)}{\leq} \underbrace{\mathscr{L}\epsilon^{\max} + \eta^*_{phy}}_{=\eta^*_R} \overset{(9)}{\leq} 0.\end{aligned}$$

Hence, $\mathcal{B}$ obtained by solving the SOP$_{phy}$ in (7) is a BC for Λ with a certain correctness guarantee, thereby completing the proof. ■

Proof [Theorem 4] As outlined in (Mohajerin Esfahani et al., 2014, Theorem 3.6 and Remark 3.9), one can formally lower bound the probabilistic difference between the optimal values of the ROP and SOP$_{phy}$ as

$$\mathbb{P}\Big\{0 \leq \eta^*_R - \eta^*_{phy} \leq Lg(\varphi)\Big\} \geq 1 - \beta, \tag{12}$$

where $g(\varphi)$ represents a uniform level-set bound, defined as $g(\varphi) = \mathscr{L}\mu^{-1}(\varphi)$ (Mohajerin Esfahani et al., 2014, Definition 3.1), while L is the Slater constant, as defined in (Mohajerin Esfahani et al., 2014, Eq. (5)). It is clear that η^*_R is always greater than or equal to η^*_{phy}, since η^*_R is determined using an infinite number of constraints, while η^*_{phy} is calculated using only a finite number of P. According to (Mohajerin Esfahani et al., 2014, Remark 3.5), since the original ROP in (3) can be formulated as a *min–max* optimization problem, the Slater constant L is chosen to be 1. Then from (12), it can be concluded that:

$$\eta^*_{phy} \leq \eta^*_R \leq \eta^*_{phy} + Lg(\varphi) \leq \eta^*_{phy} + \mathscr{L}\mu^{-1}(\varphi),$$

with a confidence of at least $1 - \beta$. Given that $\eta^*_{phy} + \mathscr{L}\mu^{-1}(\varphi) \leq 0$ according to (10), one can deduce that $\eta^*_R \leq 0$, thereby completing the proof. ■